Lorentz covariance of the canonical perfect lens

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Abstract

The canonical perfect lens — comprising three slabs, each made of a linear, homogeneous, bianisotropic material with orthorhombic symmetry — is Lorentz covariant.

Keywords: Negative refraction, anti–vacuum, vacuum, orthorhombic materials

1 Introduction

The electromagnetic properties of classical vacuum (i.e., empty space) are characterized by its permittivity $\epsilon_0 = 8.854 \times 10^{-12}\,\text{F m}^{-1}$ and permeability $\mu_0 = 4\pi \times 10^{-7}\,\text{H m}^{-1}$. In contrast, *anti–vacuum* has permittivity $-\epsilon_0$ and permeability $-\mu_0$ [1]. The perfect lens, as conceptualized by Pendry [2], consists of a slab of anti–vacuum sandwiched by two slabs of vacuum. While this perfect lens is an idealization which can never be fully realized in practice [3], the concept of the perfect lens has spawned much theoretical and experimental work within the past few years on negative refraction and metamaterials. Indeed, interest in this area continues to escalate, with negatively refracting metamaterials having now entered the visible frequency regime [4]. Aside from metamaterials, negative refraction occurs in biological scenarios [5], and there is the possibility of negative refraction arising in special [6] and general [7] relativistic scenarios.

A fundamental characteristic of vacuum is that its constitutive parameters are invariant under a Lorentz transformation. A straightforward derivation reveals that the constitutive parameters of anti–vacuum are also invariant under a Lorentz transformation. Therefore, the vacuum/anti–vacuum/vacuum perfect lens is Lorentz covariant. A canonical formulation for the perfect lens has also been developed, wherein the two constituent materials are linear, homogeneous, orthorhombic materials [3, 8]. In this Letter, we address the question: is this canonical perfect lens invariant under a Lorentz transformation?

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2   Canonical perfect lens

The canonical perfect lens comprises a slab of material labeled b, sandwiched between two slabs of a material labeled a, as schematically illustrated in Figure 1. With respect to an inertial frame of reference Σ, material a occupies the regions $0 \leq z < d_1$ and $d_1 + d_2 \leq z < d_1 + d_2 + d_3$, while material b occupies the region $d_1 \leq z < d_1 + d_2$. Both materials move with a common and uniform velocity $\mathbf{v} = v\hat{\mathbf{x}}$, where $v \in (-c_0, c_0)$ and c_0 is the speed of light in vacuum; thus, the direction of relative motion is parallel to the interfaces between material a and material b.

The materials a and b are linear and homogeneous. With respect to an inertial reference frame $\tilde{\Sigma}$ that also moves at velocity $\mathbf{v}$ relative to Σ, their frequency–domain constitutive relations are

$$
\left.
\begin{aligned}
\tilde{\mathbf{D}}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\omega}) &= \epsilon_0 \left[\underline{\underline{\tilde{\epsilon}}}^{a,b} \bullet \tilde{\mathbf{E}}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\omega}) + \underline{\underline{\tilde{\alpha}}}^{a,b} \bullet \tilde{\mathbf{H}}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\omega}) \right] \\
\tilde{\mathbf{B}}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\omega}) &= \mu_0 \left[\underline{\underline{\tilde{\beta}}}^{a,b} \bullet \tilde{\mathbf{E}}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\omega}) + \underline{\underline{\tilde{\mu}}}^{a,b} \bullet \tilde{\mathbf{H}}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\omega}) \right]
\end{aligned}
\right\}, \tag{1}
$$

wherein the 3×3 constitutive dyadics have the orthorhombic form

$$
\underline{\underline{\tilde{\xi}}}^{a,b} = \begin{bmatrix} \tilde{\xi}_{11}^{a,b} & 0 & 0 \\ 0 & \tilde{\xi}_{22}^{a,b} & 0 \\ 0 & 0 & \tilde{\xi}_{33}^{a,b} \end{bmatrix}, \qquad (\xi = \epsilon, \alpha, \beta, \mu). \tag{2}
$$

With respect to Σ, their frequency–domain constitutive relations are given as [9]

$$
\left.
\begin{aligned}
\mathbf{D}(x, y, z, \omega) &= \epsilon_0 \left[\underline{\underline{\epsilon}}^{a,b} \bullet \mathbf{E}(x, y, z, \omega) + \underline{\underline{\alpha}}^{a,b} \bullet \mathbf{H}(x, y, z, \omega) \right] \\
\mathbf{B}(x, y, z, \omega) &= \mu_0 \left[\underline{\underline{\beta}}^{a,b} \bullet \mathbf{E}(x, y, z, \omega) + \underline{\underline{\mu}}^{a,b} \bullet \mathbf{H}(x, y, z, \omega) \right]
\end{aligned}
\right\}, \tag{3}
$$

wherein the 3×3 constitutive dyadics have the form

$$
\underline{\underline{\xi}}^{a,b} = \begin{bmatrix} \xi_{11}^{a,b} & 0 & 0 \\ 0 & \xi_{22}^{a,b} & \xi_{23}^{a,b} \\ 0 & \xi_{32}^{a,b} & \xi_{33}^{a,b} \end{bmatrix}, \qquad (\xi = \epsilon, \alpha, \beta, \mu). \tag{4}
$$

Explicit expressions for the components of $\underline{\underline{\xi}}^{a,b}$, in terms of v and the components of $\underline{\underline{\tilde{\xi}}}^{a,b}$, are provided in Appendix 1.

3   Lorentz covariance

Following the approach developed for the canonical perfect lens in the co–moving reference frame $\tilde{\Sigma}$ [3, 8], we express the electromagnetic phasors $\mathbf{E}(x, y, z, \omega)$, etc., in Σ in terms of their spatial Fourier transformations with respect to x and y; thus,

$$
\left.
\begin{aligned}
\mathbf{E}(x, y, z, \omega) &= \mathbf{e}(z, \kappa, \psi, \omega) \, \exp\left[i\kappa \left(x \cos \psi + y \sin \psi \right) \right] \\
\mathbf{B}(x, y, z, \omega) &= \mathbf{b}(z, \kappa, \psi, \omega) \, \exp\left[i\kappa \left(x \cos \psi + y \sin \psi \right) \right] \\
\mathbf{D}(x, y, z, \omega) &= \mathbf{d}(z, \kappa, \psi, \omega) \, \exp\left[i\kappa \left(x \cos \psi + y \sin \psi \right) \right] \\
\mathbf{H}(x, y, z, \omega) &= \mathbf{h}(z, \kappa, \psi, \omega) \, \exp\left[i\kappa \left(x \cos \psi + y \sin \psi \right) \right]
\end{aligned}
\right\}. \tag{5}
$$

Thereby, wave propagation in the non–co–moving reference frame is described by the $4{\times}4$ matrix ordinary differential equations

$$\frac{d}{dz}\left[\underline{f}(z,\kappa,\psi,\omega)\right]=i\left[\underline{\underline{P}}_a(\kappa,\psi,\omega)\right]\boldsymbol{\cdot}\left[\underline{f}(z,\kappa,\psi,\omega)\right],$$
$$z\in(0,d_1)\text{ or }z\in(d_1+d_2,d_1+d_2+d_3)\,, \tag{6}$$

and

$$\frac{d}{dz}\left[\underline{f}(z,\kappa,\psi,\omega)\right]=i\left[\underline{\underline{P}}_b(\kappa,\psi,\omega)\right]\boldsymbol{\cdot}\left[\underline{f}(z,\kappa,\psi,\omega)\right],$$
$$z\in(d_1,d_1+d_2)\,, \tag{7}$$

with the column 4–vector

$$\left[\underline{f}\right]=[\mathbf{e}\boldsymbol{\cdot}\hat{\mathbf{x}},\,\mathbf{e}\boldsymbol{\cdot}\hat{\mathbf{y}},\,\mathbf{h}\boldsymbol{\cdot}\hat{\mathbf{x}},\,\mathbf{h}\boldsymbol{\cdot}\hat{\mathbf{y}}]^T\,. \tag{8}$$

Explicit expressions for the components of the $4{\times}4$ matrixes $\left[\underline{\underline{P}}_{a,b}\right]$ are provided in Appendix 2.

By solving (6) and (7), we see that the phasors at $z=0$ and $z=d_1+d_2+d_3$ are related as

$$\begin{aligned}\left[\underline{f}(d_1+d_2+d_3,\kappa,\psi,\omega)\right]&=\exp\left\{id_3\left[\underline{\underline{P}}_a(\kappa,\psi,\omega)\right]\right\}\boldsymbol{\cdot}\exp\left\{id_2\left[\underline{\underline{P}}_b(\kappa,\psi,\omega)\right]\right\}\\&\quad\boldsymbol{\cdot}\exp\left\{id_1\left[\underline{\underline{P}}_a(\kappa,\psi,\omega)\right]\right\}\boldsymbol{\cdot}\left[\underline{f}(0,\kappa,\psi,\omega)\right].\end{aligned} \tag{9}$$

As described elsewhere [3, 8], the solution of the problem of the canonical perfect lens involves finding the thicknesses d_1 and d_3 for material a, and the thickness d_2 for material b, such that

$$\left[\underline{f}(0,\kappa,\psi,\omega)\right]=\left[\underline{f}(d_1+d_2+d_3,\kappa,\psi,\omega)\right] \tag{10}$$

for all κ, ψ and ω.

The apparently simplest route to satisfying the perfect-lens condition (10) is to ensure that the matrixes $\underline{\underline{P}}_a(\kappa,\psi,\omega)$ and $\underline{\underline{P}}_b(\kappa,\psi,\omega)$ commute for all κ, ψ and ω. Then,

$$\underline{\underline{P}}_b(\kappa,\psi,\omega)+\gamma\underline{\underline{P}}_a(\kappa,\psi,\omega)=\underline{\underline{0}}, \tag{11}$$

and

$$d_1+d_3=\gamma d_2\,, \tag{12}$$

where $\gamma>0$ is some scalar. A straightforward calculation reveals that (11) holds for the reference frame Σ when

$$\left.\begin{aligned}\tilde{\xi}_{11}^b+\gamma\tilde{\xi}_{11}^a&=0\\\tilde{\xi}_{22}^b+\gamma\tilde{\xi}_{22}^a&=0\\\tilde{\xi}_{33}^b+\gamma^{-1}\tilde{\xi}_{33}^a&=0\end{aligned}\right\}, \qquad (\xi=\epsilon,\alpha,\beta,\mu). \tag{13}$$

In particular, since the conditions (12) and (13) hold for arbitrary $v\in(-c_0,c_0)$, the canonical perfect lens is Lorentz covariant.

Thus, not only is a perfect lens comprising slabs of vacuum, anti–vacuum, and vacuum Lorentz covariant, but combinations of linear, homogeneous, and orthorhombic mediums a and b can be found such that a perfect lens made thereof is also Lorentz covariant. The consequences of this result for space exploration and observational astronomy are matters for future consideration.

Appendix 1

The components of the 3×3 constitutive dyadics in Σ are provided by a straightforward, but cumbersome, application of the Lorentz transformation to the constitutive dyadics in $\tilde{\Sigma}$ [9]. Thus,

$$\epsilon_{11}^{a,b} = \tilde{\epsilon}_{11}^{a,b}, \tag{14}$$

$$\epsilon_{22}^{a,b} = -\frac{c_0^2 - v^2}{\Delta}\left[c_0^2\tilde{\epsilon}_{22}^{a,b} + v^2\tilde{\epsilon}_{33}^{a,b}\left(\tilde{\alpha}_{22}^{a,b}\tilde{\beta}_{22}^{a,b} - \tilde{\epsilon}_{22}^{a,b}\tilde{\mu}_{22}^{a,b}\right)\right]\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}, \tag{15}$$

$$\epsilon_{23}^{a,b} = -\frac{c_0^2 - v^2}{\Delta}\left(\tilde{\epsilon}_{33}^{a,b}\tilde{\alpha}_{22}^{a,b}\epsilon_0 - \tilde{\epsilon}_{22}^{a,b}\tilde{\beta}_{33}^{a,b}\mu_0\right)vc_0^2\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}, \tag{16}$$

$$\epsilon_{32}^{a,b} = \frac{c_0^2 - v^2}{\Delta}\left(\tilde{\epsilon}_{22}^{a,b}\tilde{\alpha}_{33}^{a,b}\epsilon_0 - \tilde{\epsilon}_{33}^{a,b}\tilde{\beta}_{22}^{a,b}\mu_0\right)vc_0^2\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}, \tag{17}$$

$$\epsilon_{33}^{a,b} = -\frac{c_0^2 - v^2}{\Delta}\left[c_0^2\tilde{\epsilon}_{33}^{a,b} + v^2\tilde{\epsilon}_{22}^{a,b}\left(\tilde{\alpha}_{33}^{a,b}\tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b}\tilde{\mu}_{33}^{a,b}\right)\right]\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}, \tag{18}$$

$$\alpha_{11}^{a,b} = \tilde{\alpha}_{11}^{a,b}, \tag{19}$$

$$\alpha_{22}^{a,b} = -\frac{c_0^2 - v^2}{\Delta}\left[c_0^2\tilde{\alpha}_{22}^{a,b}\epsilon_0 + v^2\tilde{\beta}_{33}^{a,b}\mu_0\left(\tilde{\alpha}_{22}^{a,b}\tilde{\beta}_{22}^{a,b} - \tilde{\epsilon}_{22}^{a,b}\tilde{\mu}_{22}^{a,b}\right)\right]\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}\mu_0 c_0^2, \tag{20}$$

$$\alpha_{23}^{a,b} = -\frac{vc_0^4\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}}{\Delta}\left(\mu_0\left(1 - \tilde{\epsilon}_{22}^{a,b}\tilde{\mu}_{33}^{a,b} + \mu_0^2 v^2\tilde{\beta}_{22}^{a,b}\tilde{\beta}_{33}^{a,b}\right) + \epsilon_0\left\{\tilde{\alpha}_{22}^{a,b}\tilde{\alpha}_{33}^{a,b}\right.\right.$$
$$\left.\left. + v^2\left[\tilde{\alpha}_{22}^{a,b}\tilde{\beta}_{22}^{a,b}\left(\tilde{\alpha}_{33}^{a,b}\tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b}\tilde{\mu}_{33}^{a,b}\right) - \tilde{\mu}_{22}^{a,b}\mu_0^2\left(\tilde{\alpha}_{33}^{a,b}\tilde{\beta}_{33}^{a,b}\tilde{\epsilon}_{22}^{a,b} + \tilde{\epsilon}_{33}^{a,b} - \tilde{\epsilon}_{22}^{a,b}\tilde{\epsilon}_{33}^{a,b}\tilde{\mu}_{33}^{a,b}\right)\right]\right\}\right), \tag{21}$$

$$\alpha_{32}^{a,b} = \frac{vc_0^4\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}}{\Delta}\left(\mu_0\left(1 - \tilde{\epsilon}_{33}^{a,b}\tilde{\mu}_{22}^{a,b} + \mu_0^2 v^2\tilde{\beta}_{22}^{a,b}\tilde{\beta}_{33}^{a,b}\right) + \epsilon_0\left\{\tilde{\alpha}_{22}^{a,b}\tilde{\alpha}_{33}^{a,b}\right.\right.$$
$$\left.\left. + v^2\left[\tilde{\alpha}_{22}^{a,b}\tilde{\beta}_{22}^{a,b}\left(\tilde{\alpha}_{33}^{a,b}\tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b}\tilde{\mu}_{33}^{a,b}\right) - \tilde{\epsilon}_{22}^{a,b}\mu_0^2\left(\tilde{\alpha}_{33}^{a,b}\tilde{\beta}_{33}^{a,b}\tilde{\mu}_{22}^{a,b} + \tilde{\mu}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b}\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}\right)\right]\right\}\right), \tag{22}$$

$$\alpha_{33}^{a,b} = -\frac{c_0^2 - v^2}{\Delta}\left[c_0^2\tilde{\alpha}_{33}^{a,b}\epsilon_0 + v^2\tilde{\beta}_{22}^{a,b}\mu_0\left(\tilde{\alpha}_{33}^{a,b}\tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b}\tilde{\mu}_{33}^{a,b}\right)\right]\tilde{\mu}_{22}^{a,b}\tilde{\mu}_{33}^{a,b}\mu_0 c_0^2, \tag{23}$$

$$\beta_{11}^{a,b} = \tilde{\beta}_{11}^{a,b}, \tag{24}$$

$$\beta_{22}^{a,b} = -\frac{c_0^2 - v^2}{\Delta} \left[c_0^2 \tilde{\beta}_{22}^{a,b} \mu_0 + v^2 \tilde{\alpha}_{33}^{a,b} \epsilon_0 \left(\tilde{\alpha}_{22}^{a,b} \tilde{\beta}_{22}^{a,b} - \tilde{\epsilon}_{22}^{a,b} \tilde{\mu}_{22}^{a,b} \right) \right] \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b} \epsilon_0 c_0^2, \tag{25}$$

$$\beta_{23}^{a,b} = -\frac{v c_0^4 \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b}}{\Delta} \left[\mu_0 \tilde{\beta}_{22}^{a,b} \tilde{\beta}_{33}^{a,b} + \epsilon_0 \left(1 - \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{22}^{a,b} + \epsilon_0 v^2 \left\{ \epsilon_0 \tilde{\alpha}_{22}^{a,b} \tilde{\alpha}_{33}^{a,b} \right. \right. \right.$$
$$\left. \left. \left. + \mu_0 \left[\tilde{\alpha}_{22}^{a,b} \tilde{\beta}_{22}^{a,b} \left(\tilde{\alpha}_{33}^{a,b} \tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{33}^{a,b} \right) - \tilde{\epsilon}_{22}^{a,b} \left(\tilde{\alpha}_{33}^{a,b} \tilde{\beta}_{33}^{a,b} \tilde{\mu}_{22}^{a,b} + \tilde{\mu}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b} \right) \right] \right\} \right) \right], \tag{26}$$

$$\beta_{32}^{a,b} = \frac{v c_0^4 \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b}}{\Delta} \left[\mu_0 \tilde{\beta}_{22}^{a,b} \tilde{\beta}_{33}^{a,b} + \epsilon_0 \left(1 - \tilde{\epsilon}_{22}^{a,b} \tilde{\mu}_{33}^{a,b} + \epsilon_0 v^2 \left\{ \epsilon_0 \tilde{\alpha}_{22}^{a,b} \tilde{\alpha}_{33}^{a,b} \right. \right. \right.$$
$$\left. \left. \left. + \mu_0 \left[\tilde{\alpha}_{22}^{a,b} \tilde{\beta}_{22}^{a,b} \left(\tilde{\alpha}_{33}^{a,b} \tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{33}^{a,b} \right) - \tilde{\mu}_{22}^{a,b} \left(\tilde{\alpha}_{33}^{a,b} \tilde{\beta}_{33}^{a,b} \tilde{\epsilon}_{22}^{a,b} + \tilde{\epsilon}_{33}^{a,b} - \tilde{\epsilon}_{22}^{a,b} \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{33}^{a,b} \right) \right] \right\} \right) \right], \tag{27}$$

$$\beta_{33}^{a,b} = -\frac{c_0^2 - v^2}{\Delta} \left[c_0^2 \tilde{\beta}_{33}^{a,b} \mu_0 + v^2 \tilde{\alpha}_{22}^{a,b} \epsilon_0 \left(\tilde{\alpha}_{33}^{a,b} \tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{33}^{a,b} \right) \right] \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b} \epsilon_0 c_0^2, \tag{28}$$

$$\mu_{11}^{a,b} = \tilde{\mu}_{11}^{a,b}, \tag{29}$$

$$\mu_{22}^{a,b} = -\frac{c_0^2 - v^2}{\Delta} \left[c_0^2 \tilde{\mu}_{22}^{a,b} + v^2 \tilde{\mu}_{33}^{a,b} \left(\tilde{\alpha}_{22}^{a,b} \tilde{\beta}_{22}^{a,b} - \tilde{\epsilon}_{22}^{a,b} \tilde{\mu}_{22}^{a,b} \right) \right] \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b}, \tag{30}$$

$$\mu_{23}^{a,b} = -\frac{c_0^2 - v^2}{\Delta} \left(\tilde{\alpha}_{33}^{a,b} \tilde{\mu}_{22}^{a,b} \epsilon_0 - \tilde{\beta}_{22}^{a,b} \tilde{\mu}_{33}^{a,b} \mu_0 \right) v c_0^2 \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b}, \tag{31}$$

$$\mu_{32}^{a,b} = \frac{c_0^2 - v^2}{\Delta} \left(\tilde{\alpha}_{22}^{a,b} \tilde{\beta}_{33}^{a,b} \epsilon_0 - \tilde{\beta}_{33}^{a,b} \tilde{\mu}_{22}^{a,b} \mu_0 \right) v c_0^2 \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b}, \tag{32}$$

$$\mu_{33}^{a,b} = -\frac{c_0^2 - v^2}{\Delta} \left[c_0^2 \tilde{\mu}_{33}^{a,b} + v^2 \tilde{\mu}_{22}^{a,b} \left(\tilde{\alpha}_{33}^{a,b} \tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{33}^{a,b} \right) \right] \tilde{\mu}_{22}^{a,b} \tilde{\mu}_{33}^{a,b}, \tag{33}$$

where

$$\Delta = c_0^4 v^2 \left(\tilde{\alpha}_{22}^{a,b} \tilde{\mu}_{33}^{a,b} \epsilon_0 - \tilde{\beta}_{33}^{a,b} \tilde{\mu}_{22}^{a,b} \mu_0 \right) \left(\tilde{\beta}_{22}^{a,b} \tilde{\mu}_{33}^{a,b} \mu_0 - \tilde{\alpha}_{33}^{a,b} \tilde{\mu}_{22}^{a,b} \epsilon_0 \right)$$
$$- \left[c_0^2 \tilde{\mu}_{22}^{a,b} + v^2 \tilde{\mu}_{33}^{a,b} \left(\tilde{\alpha}_{22}^{a,b} \tilde{\beta}_{22}^{a,b} - \tilde{\epsilon}_{22}^{a,b} \tilde{\mu}_{22}^{a,b} \right) \right]$$
$$\times \left[c_0^2 \tilde{\mu}_{33}^{a,b} + v^2 \tilde{\mu}_{22}^{a,b} \left(\tilde{\alpha}_{33}^{a,b} \tilde{\beta}_{33}^{a,b} - \tilde{\epsilon}_{33}^{a,b} \tilde{\mu}_{33}^{a,b} \right) \right]. \tag{34}$$

Appendix 2

The matrix ordinary differential equation approach to solving two–point boundary–value problems, as implemented for this Letter, is described at length elsewhere [10]. The components of the 4×4 matrixes $\left[\underline{\underline{P}}_{a,b} \right]$ are delivered by a straightforward manipulation of the frequency–domain Maxwell

postulates, together with the constitutive relations (3). Thus,

$$\left[\underline{\underline{P}}_{a,b}\right]_{11} = \frac{\kappa_y}{\rho^{a,b}}\left(\frac{\kappa_x\alpha_{33}^{a,b}}{\omega\mu_0} + \alpha_{33}^{a,b}\beta_{23}^{a,b} - \epsilon_{33}^{a,b}\mu_{23}^{a,b}\right), \tag{35}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{12} = \omega\mu_0\beta_{22}^{a,b} - \frac{1}{\omega\rho^{a,b}}\left(\alpha_{33}^{a,b}\left[\frac{1}{\mu_0}\kappa_x^2 + \omega\kappa_x\left(\beta_{23}^{a,b} - \beta_{32}^{a,b}\right) - \omega^2\mu_0\beta_{23}^{a,b}\beta_{33}^{a,b}\right]\right.$$
$$\left. -\omega\left\{\mu_{23}^{a,b}\left[\omega\mu_0\epsilon_{32}^{a,b}\beta_{33}^{a,b} + \epsilon_{33}^{a,b}\left(\kappa_x - \omega\mu_0\beta_{32}^{a,b}\right)\right] - \epsilon_{32}^{a,b}\mu_{33}^{a,b}\left(\kappa_x + \omega\mu_0\beta_{23}^{a,b}\right)\right\}\right), \tag{36}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{13} = \frac{\kappa_y}{\rho^{a,b}}\left[\frac{\kappa_x\mu_{33}^{a,b}}{\omega\epsilon_0} + \frac{\mu_0\left(\mu_{33}^{a,b}\beta_{23}^{a,b} - \beta_{33}^{a,b}\mu_{23}^{a,b}\right)}{\epsilon_0}\right], \tag{37}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{14} = \omega\mu_0\mu_{22}^{a,b} - \frac{1}{\omega\rho^{a,b}}\left(\mu_{33}^{a,b}\left[\frac{1}{\epsilon_0}\kappa_x^2 + \omega\kappa_x\left(\alpha_{32}^{a,b} + \frac{\mu_0}{\epsilon_0}\beta_{23}^{a,b}\right) + \omega^2\mu_0\alpha_{32}^{a,b}\beta_{23}^{a,b}\right]\right.$$
$$\left. +\omega\left\{\mu_{23}^{a,b}\left[\omega\mu_0\epsilon_{33}^{a,b}\beta_{32}^{a,b} - \beta_{33}^{a,b}\left(\frac{\mu_0}{\epsilon_0}\kappa_x + \omega\mu_0\alpha_{32}^{a,b}\right)\right] - \epsilon_{33}^{a,b}\mu_{32}^{a,b}\left(\kappa_x + \omega\mu_0\beta_{23}^{a,b}\right)\right\}\right), \tag{38}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{21} = -\omega\mu_0\beta_{11}^{a,b} + \frac{\kappa_y^2\alpha_{33}^{a,b}}{\omega\mu_0\rho^{a,b}}, \tag{39}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{22} = -\frac{\kappa_y}{\rho^{a,b}}\left(\frac{\kappa_x\alpha_{33}^{a,b}}{\omega\mu_0} - \alpha_{33}^{a,b}\beta_{32}^{a,b} + \epsilon_{32}^{a,b}\mu_{33}^{a,b}\right), \tag{40}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{23} = -\omega\mu_0\mu_{11}^{a,b} + \frac{\kappa_y^2\mu_{33}^{a,b}}{\omega\epsilon_0\rho^{a,b}}, \tag{41}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{24} = -\frac{\kappa_y}{\rho^{a,b}}\left(\frac{\kappa_x\mu_{33}^{a,b}}{\omega\epsilon_0} - \alpha_{33}^{a,b}\mu_{32}^{a,b} + \alpha_{32}^{a,b}\mu_{33}^{a,b}\right), \tag{42}$$

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$$\left[\underline{\underline{P}}_{a,b}\right]_{31} = -\frac{\kappa_y}{\rho^{a,b}}\left[\frac{\kappa_x \epsilon_{33}^{a,b}}{\omega\mu_0} + \frac{\epsilon_0\left(\alpha_{33}^{a,b}\epsilon_{23}^{a,b} - \epsilon_{33}^{a,b}\alpha_{23}^{a,b}\right)}{\mu_0}\right], \tag{43}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{32} = -\omega\epsilon_0\epsilon_{22}^{a,b} + \frac{1}{\omega\rho^{a,b}}\left(\epsilon_{33}^{a,b}\left[\frac{1}{\mu_0}\kappa_x^2 - \omega\kappa_x\left(\frac{\epsilon_0}{\mu_0}\alpha_{23}^{a,b} + \beta_{32}^{a,b}\right) + \omega^2\epsilon_0\alpha_{23}^{a,b}\beta_{32}^{a,b}\right]\right.$$
$$\left.+\omega\left\{\epsilon_{23}^{a,b}\left[\omega\epsilon_0\epsilon_{32}^{a,b}\mu_{33}^{a,b} + \alpha_{33}^{a,b}\left(\frac{\epsilon_0}{\mu_0}\kappa_x - \omega\epsilon_0\beta_{32}^{a,b}\right)\right] + \epsilon_{32}^{a,b}\beta_{33}^{a,b}\left(\kappa_x - \omega\epsilon_0\alpha_{23}^{a,b}\right)\right\}\right), \tag{44}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{33} = -\frac{\kappa_y}{\rho^{a,b}}\left(\frac{\kappa_x\beta_{33}^{a,b}}{\omega\epsilon_0} - \alpha_{23}^{a,b}\beta_{33}^{a,b} + \epsilon_{23}^{a,b}\mu_{33}^{a,b}\right), \tag{45}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{34} = -\omega\epsilon_0\alpha_{22}^{a,b} + \frac{1}{\omega\rho^{a,b}}\left(\beta_{33}^{a,b}\left[\frac{1}{\epsilon_0}\kappa_x^2 + \omega\kappa_x\left(\alpha_{32}^{a,b} - \alpha_{23}^{a,b}\right) - \omega^2\epsilon_0\alpha_{23}^{a,b}\alpha_{32}^{a,b}\right]\right.$$
$$\left.-\omega\left\{\mu_{32}^{a,b}\left[\omega\epsilon_0\epsilon_{23}^{a,b}\alpha_{33}^{a,b} + \epsilon_{33}^{a,b}\left(\kappa_x - \omega\epsilon_0\alpha_{23}^{a,b}\right)\right] + \epsilon_{23}^{a,b}\mu_{33}^{a,b}\left(\kappa_x + \omega\epsilon_0\alpha_{32}^{a,b}\right)\right\}\right), \tag{46}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{41} = \omega\epsilon_0\epsilon_{11}^{a,b} - \frac{\kappa_y^2\epsilon_{33}^{a,b}}{\omega\mu_0\rho^{a,b}}, \tag{47}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{42} = \frac{\kappa_y}{\rho^{a,b}}\left(\frac{\kappa_x\epsilon_{33}^{a,b}}{\omega\mu_0} - \epsilon_{33}^{a,b}\beta_{32}^{a,b} + \epsilon_{32}^{a,b}\beta_{33}^{a,b}\right), \tag{48}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{43} = \omega\epsilon_0\alpha_{11}^{a,b} - \frac{\kappa_y^2\beta_{33}^{a,b}}{\omega\epsilon_0\rho^{a,b}}, \tag{49}$$

$$\left[\underline{\underline{P}}_{a,b}\right]_{44} = \frac{\kappa_y}{\rho^{a,b}}\left(\frac{\kappa_x\beta_{33}^{a,b}}{\omega\epsilon_0} + \alpha_{32}^{a,b}\beta_{33}^{a,b} - \epsilon_{33}^{a,b}\mu_{32}^{a,b}\right), \tag{50}$$

with

$$\left.\begin{array}{r}\rho^{a,b} = \epsilon_{33}^{a,b}\mu_{33}^{a,b} - \alpha_{33}^{a,b}\beta_{33}^{a,b} \\ \kappa_x = \kappa\cos\psi \\ \kappa_y = \kappa\sin\psi\end{array}\right\}. \tag{51}$$

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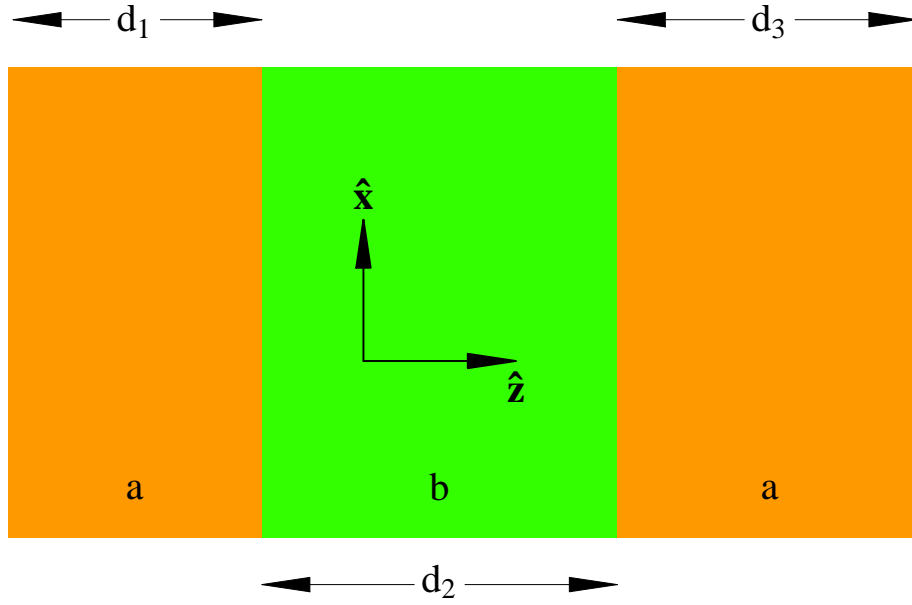[10] A. Lakhtakia, W.S. Weiglhofer, IEE Proc.–Microw. Antennas Propagat. 144 (1997) 57.

Figure 1: Schematic of the canonical perfect lens.